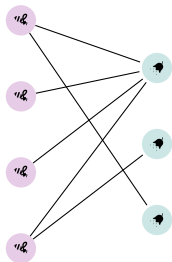


# Joint analysis of bipartite networks collection

Présentation LSD

Louis Lacoste, under the supervision of Pierre Barbillon and Sophie  
Donnet  
Laboratoire MIA Paris-Saclay

# Why a network?



$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

Associated  
adjacency  
matrix

Figure 1: Example of a network

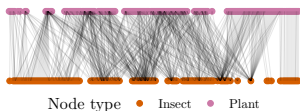


Figure 2: Plant-pollinator network of Bristol  
Baldock et al., 2019

- Modeling of various interactions, here ecosystems
- Structure necessary for: biodiversity monitoring, robustness, risk of collapse
- Increasingly available

# Analysis methods for a network

Several methods :

- Metrics : degree, centrality, nesting . . .
- Network embedding with GNN
- *Clustering* of nodes with latent variable models

# Analysis methods for a network

Several methods :

- Metrics : degree, centrality, nesting . . .
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- **Clustering of nodes with latent variable models**

# Latent Block Model (LBM<sup>1</sup>)

Govaert and Nadif, 2005.

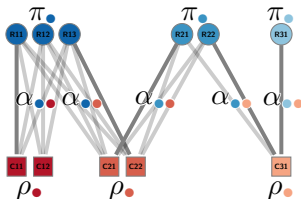


Figure 3: Example of LBM<sup>1</sup>

## Hierarchical model

$$\forall q \in \llbracket 1, Q_1 \rrbracket, \mathbb{P}(Z_i = q) = \pi_q$$

$$\forall r \in \llbracket 1, Q_2 \rrbracket, \mathbb{P}(W_j = r) = \rho_r$$

$$Y_{ij} | Z_i, W_j \sim \mathcal{F}(\alpha_{Z_i, W_j})$$

where

$$|\pi| = Q_1, |\rho| = Q_2, |\alpha| = Q_1 \times Q_2$$

## Concise LBM formula

$$Y \sim \mathcal{F}\text{-BiSBM}_{n_1, n_2}(Q_1, Q_2, \pi, \rho, \alpha)$$

<sup>1</sup>Which I will henceforth call BiSBM

# Latent Block Model (LBM<sup>1</sup>)

Govaert and Nadif, 2005.

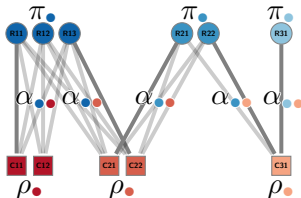


Figure 3: Example of LBM<sup>1</sup>

With

- $Q_1 = |\{\bullet, \bullet, \bullet\}|$  fixed row blocks
- $Q_2 = |\{\bullet, \bullet, \bullet\}|$  fixed column blocks

## Parameters

- $\pi_{\bullet} = \mathbb{P}(Z_i = \bullet)$
- $\rho_{\bullet} = \mathbb{P}(W_j = \bullet)$
- $\alpha_{\bullet\bullet} = \mathbb{P}(Y_{ij} = 1 | Z_i = \bullet, W_j = \bullet)$

<sup>1</sup>Which I will henceforth call BiSBM

# Multiple networks

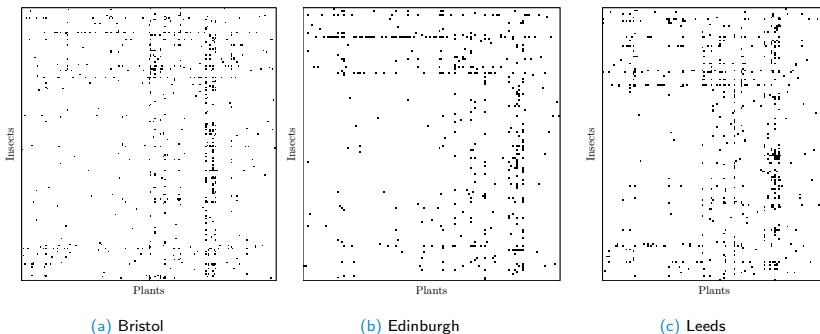


Figure 4: Adjacency matrices, Baldock et al., 2019

## Bipartite collections

$$\forall m \in \{1 \dots M\}, Y^m \stackrel{ind}{\sim} \mathcal{F}\text{-BiSBM}_{n_1^m, n_2^m}(Q_1^m, Q_2^m, \pi^m, \rho^m, \alpha^m)$$

# Bipartite collections

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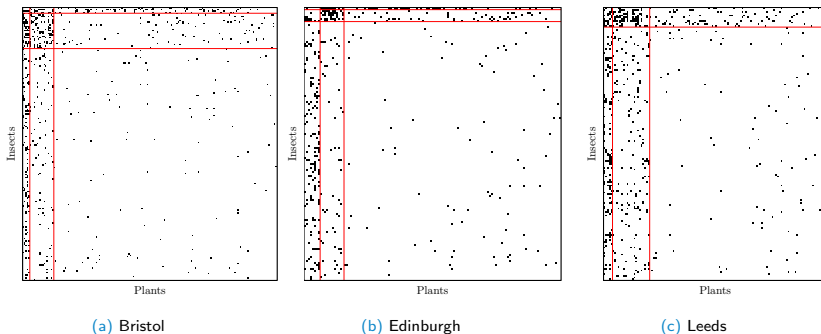


Figure 5: Reordered adjacency matrices, thanks to LBM

## Different models

### *iid*-colBiSBM

$$\forall m \in \{1 \dots M\}, Y^m \stackrel{iid}{\sim} \mathcal{F}\text{-BiSBM}_{n_1^m, n_2^m}(Q_1, Q_2, \pi, \rho, \alpha)$$

with  $\theta = (\pi, \rho, \alpha)$ .

## Different models

### *iid*-colBiSBM

$$\forall m \in \{1 \dots M\}, Y^m \stackrel{iid}{\sim} \mathcal{F}\text{-BiSBM}_{n_1^m, n_2^m}(Q_1, Q_2, \pi, \rho, \alpha)$$

with  $\theta = (\pi, \rho, \alpha)$ .

### $\pi\rho$ -colBiSBM

$$\forall m \in \{1 \dots M\}, Y^m \stackrel{ind}{\sim} \mathcal{F}\text{-BiSBM}_{n_1^m, n_2^m}(Q_1, Q_2, \pi^m, \rho^m, \alpha)$$

with  $\theta = ((\pi^m)_{m=1, \dots, M}, (\rho^m)_{m=1, \dots, M}, \alpha)$ .

# Parameter estimation

Maximizing the log-likelihood?

log-likelihood and complete log-likelihood

$$\ell(\mathbf{Y}; \theta) = \sum_{\mathbf{Z}, \mathbf{W} \in \mathcal{Z} \times \mathcal{W}} \ell_c(\mathbf{Y}, \mathbf{Z}, \mathbf{W}; \theta)$$

with  $\mathcal{Z} = \{1, \dots, Q_1\}^n$ ,  $\mathcal{W} = \{1, \dots, Q_2\}^n$

# Parameter estimation

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with  $\mathcal{Z} = \{1, \dots, Q_1\}^n$ ,  $\mathcal{W} = \{1, \dots, Q_2\}^n$

So, classic algorithm  $\Rightarrow$  *Expectation-Maximization* (EM).

## By classic EM

At iteration ( $t$ ):

- **E Step:** calculate

$$Q(\theta|\theta^{(t-1)}) = \mathbb{E}_{\mathbf{Z}, \mathbf{W}|\mathbf{Y}, \theta^{(t-1)}} [\ell_c(\mathbf{Y}, \mathbf{W}, \mathbf{Z}; \theta)]$$

- **M Step:**

$$\theta^{(t)} = \arg \max_{\theta} Q(\theta|\theta^{(t-1)})$$

## By classic EM

At iteration ( $t$ ):

- **E Step:** calculate

$$Q(\theta|\theta^{(t-1)}) = \mathbb{E}_{\mathbf{Z}, \mathbf{W}|\mathbf{Y}, \theta^{(t-1)}} [\ell_c(\mathbf{Y}, \mathbf{W}, \mathbf{Z}; \theta)]$$

- **M Step:**

$$\theta^{(t)} = \arg \max_{\theta} Q(\theta|\theta^{(t-1)})$$

### Problem for classic EM

Law of  $\mathbf{Z}, \mathbf{W}|\mathbf{Y}, \theta^{(t-1)}$  inaccessible

By *Variational EM*, as proposed by Daudin et al., 2008; Chabert-Liddell et al., 2024.

Variational approximation of  $\mathbf{Z}, \mathbf{W} | \mathbf{Y}, \theta^{(t-1)}$

$\mathcal{R}_{Y^m, \tau}(\mathbf{Z}^m, \mathbf{W}^m) = \mathcal{R}_{Y^m, \tau}^1(\mathbf{Z}^m) \times \mathcal{R}_{Y^m, \tau}^2(\mathbf{W}^m) \Rightarrow$  independence rows, columns.

$$\ell(\mathbf{Y}; \theta) \geq \sum_{m=1}^M \left( \mathcal{Q}^m(\theta \mid \theta^{(t)}) + \mathcal{H}(\mathcal{R}_{Y^m, \theta^{(t)}}(\mathbf{Z}^m, \mathbf{W}^m)) \right) =: \mathcal{J}(\tau; \theta)$$

where  $\mathcal{Q}^m(\theta \mid \theta^{(t)}) = \mathbb{E}_{\mathbf{Z}^m, \mathbf{W}^m \sim \mathcal{R}_{Y^m, \tau}(\cdot)} [\ell_c(Y^m, \mathbf{Z}^m, \mathbf{W}^m | \theta)]$

## Developed formula of variational EM

$$\begin{aligned}
 \ell(\mathbf{Y}; \theta) \geq & \sum_{m=1}^M \left( \sum_{i=1}^{n_1^m} \sum_{j=1}^{n_2^m} \sum_{q \in Q_{1,m}} \sum_{r \in Q_{2,m}} \tau_{i,q}^{1,m} \tau_{j,r}^{2,m} \log f(Y_{ij}^m; \alpha_{qr}) \right. \\
 & + \sum_{i=1}^{n_1^m} \sum_{q \in Q_{1,m}} \tau_{i,q}^{1,m} \log \pi_q^m + \sum_{j=1}^{n_2^m} \sum_{r \in Q_{2,m}} \tau_{j,r}^{2,m} \log \rho_r^m \\
 & \left. - \sum_{i=1}^{n_1} \tau_{i,q}^{1,m} \log \tau_{i,q}^{1,m} - \sum_{j=1}^{n_2} \tau_{j,r}^{2,m} \log \tau_{j,r}^{2,m} \right) =: \mathcal{J}(\tau; \theta),
 \end{aligned}$$

### Variational approximation

$$\tau_{iq}^{1,m} = \mathcal{R}_{Y^m, \tau}^1(Z_{iq}^m = 1) \text{ and } \tau_{jr}^{2,m} = \mathcal{R}_{Y^m, \tau}^2(W_{jr}^m = 1)$$

# Variational Expectation Step

$$\hat{\tau}^{(t+1)} = \arg \max_{\tau} \mathcal{J}(\tau, \hat{\theta}^{(t)}) \Leftrightarrow \arg \min_{\tau \in \mathcal{T}} \mathbf{KL}[\mathcal{R}_{\mathbf{Y}, \tau}, \mathbb{P}(.|\mathbf{Y})]$$

$$\begin{cases} \hat{\tau}_{iq}^{1,m} \propto \hat{\pi}_q^{m(t)} \prod_{j=1}^{n_2^m} \prod_{r \in \mathcal{Q}_2^m} f(Y_{ij}^m; \hat{\alpha}_{qr}^{(t)})^{\hat{\tau}_{jr}^{2,m(t+1)}} & \forall i = 1, \dots, n_1^m, q \in \mathcal{Q}_1^m \\ \hat{\tau}_{jr}^{2,m} \propto \hat{\rho}_r^{m(t)} \prod_{i=1}^{n_1^m} \prod_{q \in \mathcal{Q}_1^m} f(Y_{ij}^m; \hat{\alpha}_{qr}^{(t)})^{\hat{\tau}_{iq}^{1,m(t+1)}} & \forall j = 1, \dots, n_2^m, r \in \mathcal{Q}_2^m \end{cases}$$

---

<sup>2</sup>Initialization of  $\hat{\tau}$  with a *spectral clustering* on the networks.

## Maximization Step

$$\hat{\theta}^{(t+1)} = \arg \max_{\theta} \mathcal{J}(\hat{\tau}^{(t+1)}, \theta)$$

### Connectivity parameters

$$\hat{\alpha}_{qr} = \frac{\sum_{m=1}^M \sum_{i=1}^{n_1^m} \sum_{j=1}^{n_2^m} \tau_{iq}^{1,m} \tau_{jr}^{2,m} Y_{ij}^m}{\sum_{m=1}^M \sum_{i=1}^{n_1^m} \sum_{j=1}^{n_2^m} \tau_{iq}^{1,m} \tau_{jr}^{2,m}}$$

### Proportions for *iid*

$$\hat{\pi}_q = \frac{\sum_{m=1}^M \sum_{i=1}^{n_1^m} \tau_{iq}^{1,m}}{\sum_{m=1}^M n_1^m}$$

$$\hat{\rho}_r = \frac{\sum_{m=1}^M \sum_{j=1}^{n_2^m} \tau_{jr}^{2,m}}{\sum_{m=1}^M n_2^m}$$

## Maximization Step

$$\hat{\theta}^{(t+1)} = \arg \max_{\theta} \mathcal{J}(\hat{\tau}^{(t+1)}, \theta)$$

### Connectivity parameters

$$\hat{\alpha}_{qr} = \frac{\sum_{m=1}^M \sum_{i=1}^{n_1^m} \sum_{j=1}^{n_2^m} \tau_{iq}^{1,m} \tau_{jr}^{2,m} \mathbf{Y}_{ij}^m}{\sum_{m=1}^M \sum_{i=1}^{n_1^m} \sum_{j=1}^{n_2^m} \tau_{iq}^{1,m} \tau_{jr}^{2,m}}$$

### Proportions for $\pi\rho$

$$\hat{\pi}^m_q = \frac{\sum_{i=1}^{n_1^m} \tau_{iq}^{1,m}}{n_1^m}$$

$$\hat{\rho}^m_r = \frac{\sum_{j=1}^{n_2^m} \tau_{jr}^{2,m}}{n_2^m}$$

## Problem of choosing $(Q_1, Q_2)$

Need to select  $Q_1$  and  $Q_2$ . BIC-Like criterion<sup>2</sup>

$$\begin{aligned}\text{BIC-L}(\mathbf{Y}, Q_1, Q_2) &= \max_{\theta} \mathbb{E}_{\mathcal{R}_{\mathbf{Y}, \hat{\tau}}} [\ell_c(\mathbf{Y}, \mathbf{Z}, \mathbf{W}; \theta)] + \mathcal{H}(\mathcal{R}_{\mathbf{Y}, \hat{\tau}}) - \frac{1}{2} \text{pen}(\theta, Q_1, Q_2) \\ &= \max_{\theta} \mathcal{J}(\mathcal{R}_{\mathbf{Y}, \hat{\tau}}, \theta) - \frac{1}{2} \text{pen}(\theta, Q_1, Q_2)\end{aligned}$$

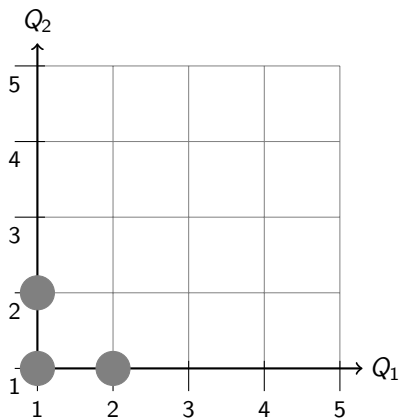
### Exploration problems

- Exploration of  $\mathbb{N}^2$  costly.
- Sensitivity to initializations.

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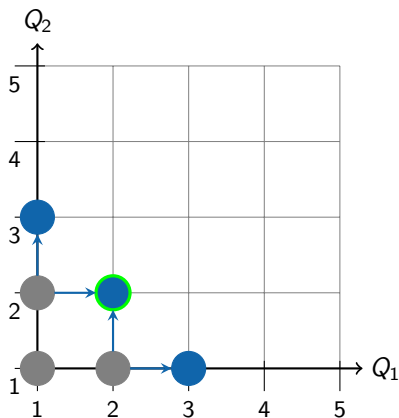
<sup>2</sup>ICL + Entropy + penalty

## Choice of $(Q_1, Q_2)$ - Greedy approach



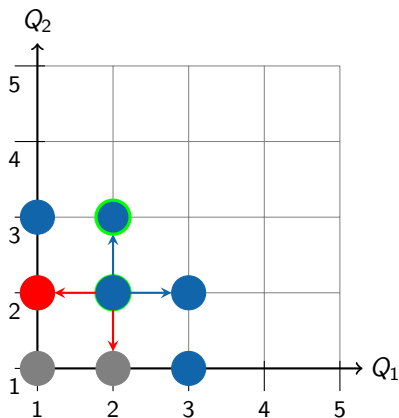
- Initial model :





## Choice of $(Q_1, Q_2)$ - Greedy approach



- Initial model :  
●
- Model after *split* :  
●
- Model maximizing the criterion :  
○

## Choice of $(Q_1, Q_2)$ - Greedy approach



- Initial model :  

- Model after *split* :  

- Model maximizing the criterion :  

- Model after *merge* :  


# Choice of $(Q_1, Q_2)$ - Sliding window

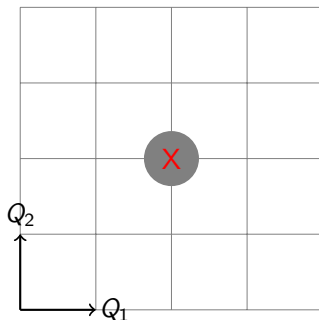


Figure 6: Sliding window

# Choice of $(Q_1, Q_2)$ - Sliding window

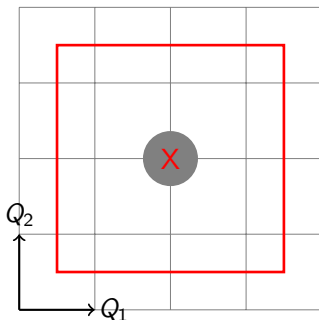


Figure 6: Sliding window

## Choice of $(Q_1, Q_2)$ - Sliding window

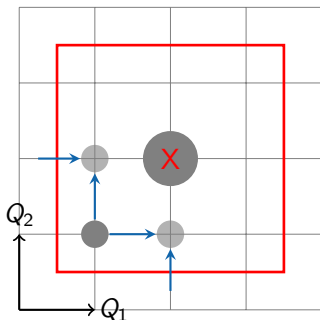


Figure 6: Sliding window

Initialization of the model if necessary

## Choice of $(Q_1, Q_2)$ - Sliding window

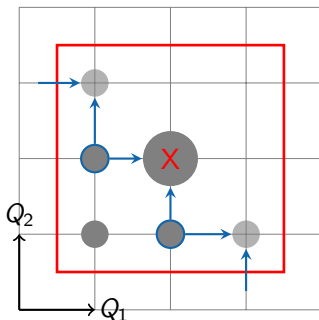


Figure 6: Sliding window

# Choice of $(Q_1, Q_2)$ - Sliding window

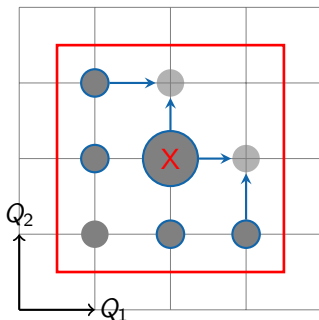


Figure 6: Sliding window

## Choice of $(Q_1, Q_2)$ - Sliding window

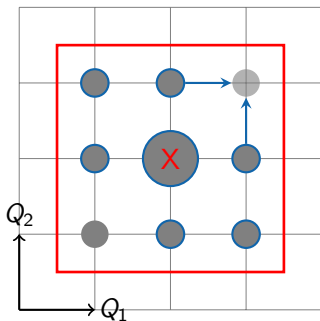


Figure 6: Sliding window

# Choice of $(Q_1, Q_2)$ - Sliding window

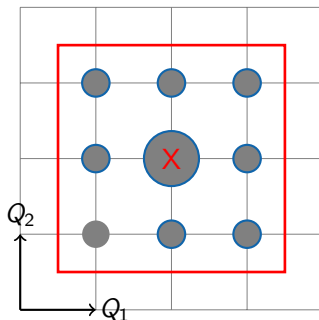


Figure 6: Sliding window

## Choice of $(Q_1, Q_2)$ - Sliding window

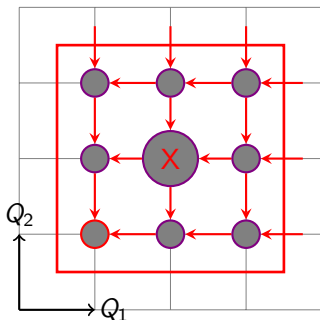


Figure 6: Sliding window

# Choice of $(Q_1, Q_2)$ - Sliding window

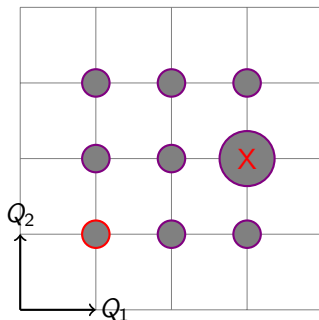


Figure 6: Sliding window

Localization of the new mode

## Choice of $(Q_1, Q_2)$ - Sliding window

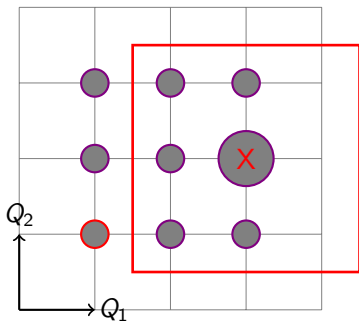
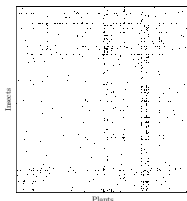


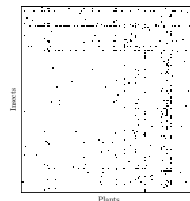
Figure 6: Sliding window

Move to the new mode then iterate

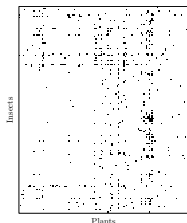
# Results Baldock et al., 2019



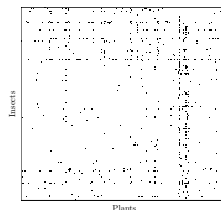
(a) Bristol



(b) Edinburgh



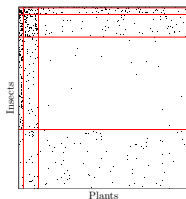
(c) Leeds



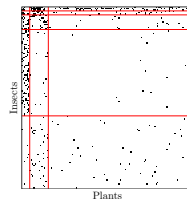
(d) Reading

Figure 7: Adjacency matrices, Baldock et al., 2019

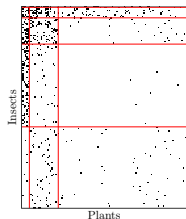
# Results Baldock et al., 2019



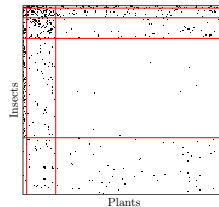
(a) Bristol



(b) Edinburgh



(c) Leeds



(d) Reading

Figure 7: Reordered adjacency matrices by *iid*-colBiSBM, Baldock et al., 2019

# Network clustering

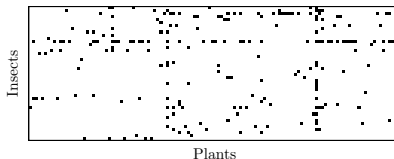
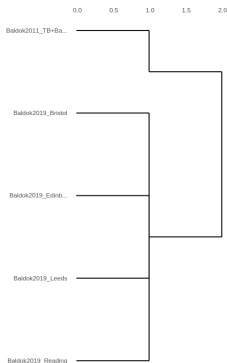
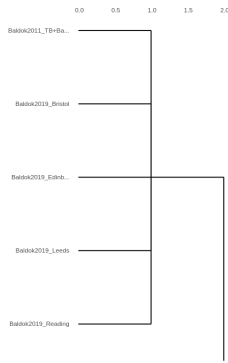


Figure 8: Adjacency matrix, Baldock et al., 2011

# Application to Baldock et al., 2019, 2011 I



(a) Model  $iid$



(b) Model  $\pi\rho$

Figure 9: Partitioning of networks of Baldock et al., 2019, 2011

# Application to Baldock et al., 2019, 2011 II

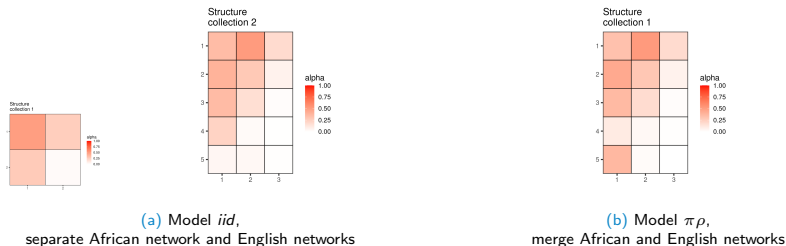
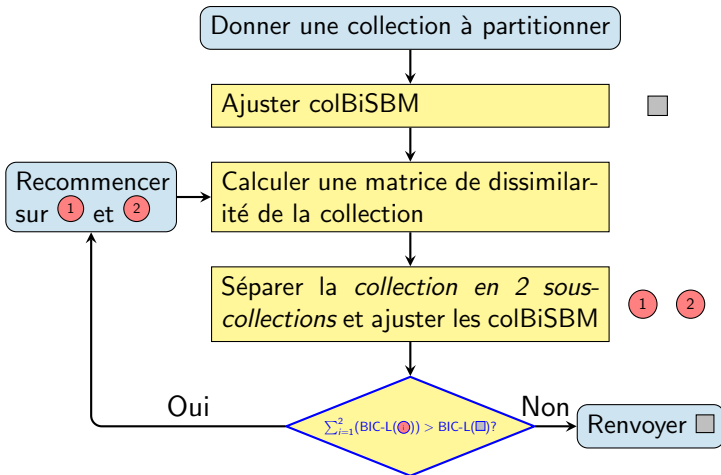


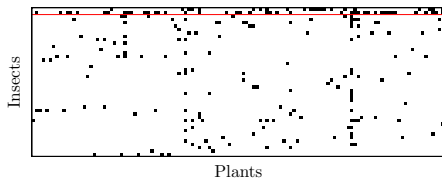
Figure 10: Structures detected for networks of Baldock et al., 2019, 2011

# Clustering algorithm

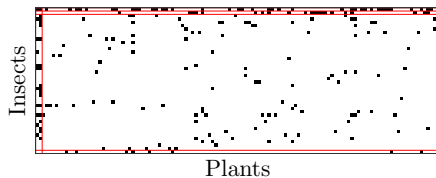


$$D_{\mathcal{M}}(m, m') = \sum_{q=1}^{Q_1} \sum_{r=1}^{Q_2} \max(\tilde{\pi}_q^m, \tilde{\pi}_q^{m'}) \left( \tilde{\alpha}_{qr}^m - \tilde{\alpha}_{qr}^{m'} \right)^2 \max(\tilde{\rho}_r^m, \tilde{\rho}_r^{m'})$$

# Results



(a) Reordered by LBM



(b) Reordered by  $\pi\rho$ -colBiSBM

Figure 11: Reordered adjacency matrix by  $\pi\rho$ -colBiSBM, Baldock et al., 2011

# Conclusion and perspectives

## Capabilities

- 4 models including 3 with flexibility on at least one of the dimensions (adaptability to data).
- Detect classic and less classic structures in an agnostic way.
- Partition a set of networks according to their structures.

# Perspectives

- Investigate stability against randomness and local *optima*.
- Proof of identifiability of the  $\pi\rho$  model.

## Package and applications

- Integration into the colSBM package, improvement of user interface and addition of ecologists' feedback
- CRAN publication
- Integrate the possibility of an additional criterion for clustering
- Apply clustering to data from [Pichon et al., 2024](#); [Doré et al., 2021](#)

Thank you for your attention !

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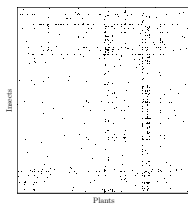
## Why does VE minimizes KL ?

$$\begin{aligned}
 \ell_c(\mathbf{Y}, \mathbf{Z}, \mathbf{W}; \theta) &= \log \mathbb{P}(\mathbf{Z}, \mathbf{W} | \mathbf{Y}; \theta) + \ell(\mathbf{Y}; \theta) \\
 &\Leftrightarrow \ell(\mathbf{Y}; \theta) = \ell_c(\mathbf{Y}, \mathbf{Z}, \mathbf{W}; \theta) - \log \mathbb{P}(\mathbf{Z}, \mathbf{W} | \mathbf{Y}; \theta) \\
 &\Leftrightarrow \mathbb{E}_{\mathcal{R}_{\mathbf{Y}, \tau}}[\ell(\mathbf{Y}; \theta)] = \mathbb{E}_{\mathcal{R}_{\mathbf{Y}, \tau}}[\ell_c(\mathbf{Y}, \mathbf{Z}, \mathbf{W}; \theta)] - \mathbb{E}_{\mathcal{R}_{\mathbf{Y}, \tau}}[\log \mathbb{P}(\mathbf{Z}, \mathbf{W} | \mathbf{Y}; \theta)] \\
 &\Leftrightarrow \ell(\mathbf{Y}; \theta) = \mathbb{E}_{\mathcal{R}_{\mathbf{Y}, \tau}}[\ell_c(\mathbf{Y}, \mathbf{Z}, \mathbf{W}; \theta)] - \mathbb{E}_{\mathcal{R}_{\mathbf{Y}, \tau}}[\log \mathbb{P}(\mathbf{Z}, \mathbf{W} | \mathbf{Y}; \theta)]
 \end{aligned}$$

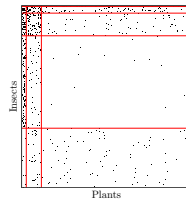
$$\begin{aligned}
 \text{But } \mathbf{KL}[\mathcal{R}_{\mathbf{Y}, \tau}, \log \mathbb{P}(\mathbf{Z}, \mathbf{W} | \mathbf{Y}; \theta)] &= -\mathbb{E}_{\mathcal{R}_{\mathbf{Y}, \tau}}\left[\log \frac{\mathbb{P}(\mathbf{Z}, \mathbf{W} | \mathbf{Y}; \theta)}{\mathcal{R}_{\mathbf{Y}, \tau}}\right] \\
 &= -\mathbb{E}_{\mathcal{R}_{\mathbf{Y}, \tau}}[\log \mathbb{P}(\mathbf{Z}, \mathbf{W} | \mathbf{Y}; \theta)] + \underbrace{\mathbb{E}_{\mathcal{R}_{\mathbf{Y}, \tau}}[\log \mathcal{R}_{\mathbf{Y}, \tau}]}_{-\mathcal{H}(\mathcal{R}_{\mathbf{Y}, \tau})}
 \end{aligned}$$

$$\Leftrightarrow \mathbf{KL}[\mathcal{R}_{\mathbf{Y}, \tau}, \log \mathbb{P}(\mathbf{Z}, \mathbf{W} | \mathbf{Y}; \theta)] + \mathcal{H}(\mathcal{R}_{\mathbf{Y}, \tau}) = -\mathbb{E}_{\mathcal{R}_{\mathbf{Y}, \tau}}[\log \mathbb{P}(\mathbf{Z}, \mathbf{W} | \mathbf{Y}; \theta)]$$

Thus  $\ell(\mathbf{Y}; \theta) - \mathbf{KL}[\mathcal{R}_{\mathbf{Y}, \tau}, \log \mathbb{P}(\mathbf{Z}, \mathbf{W} | \mathbf{Y}; \theta)] = \mathcal{J}(\tau; \theta)$   $\square$

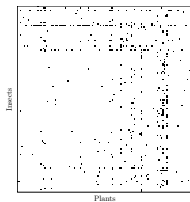


(a) Donnée

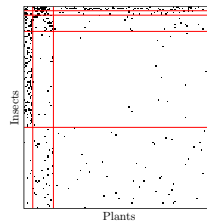


(b) Reordered

Figure 12: Bristol

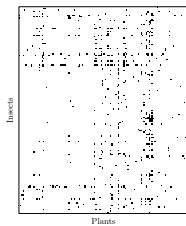


(a) Donnée

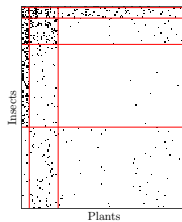


(b) Reordered

Figure 13: Edinburgh

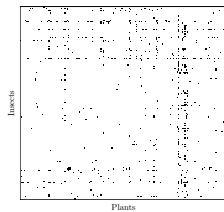


(a) Donnée

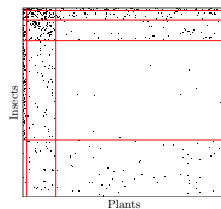


(b) Réordonnée

Figure 14: Leeds



(a) Donnée



(b) Réordonnée

Figure 15: Reading

## Appendices references I